

MATH 105A, Real Analysis, Summer Session 2014.

MWF 9:30 am – 12:00 noon, Earth and Marine Sciences, B210.

<http://people.ucsc.edu/~yorik/105A>

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**Office Hours:** Thursday, 12:00 – 2:00 pm, or by appointment.

**Required text:** *Principles of Mathematical Analysis*, 3<sup>rd</sup> edition, by Walter Rudin.

**Course Description:** This course provides an introduction to the foundations of real analysis. Topics include the topology of the real line and Euclidean space, sequences and series, and the properties of continuous and differentiable functions. See lecture schedule for more details.

**Exams:** There will be four short exams and a comprehensive final exam. The exam dates are listed in the lecture schedule that follows.

**Reading:** The assigned reading from Rudin's book (on the lecture schedule that follows) should be completed at least once **before** the corresponding lecture, and repeated until you fully grasp the material.

**Homework:** Homework will be assigned on the syllabus and is meant to help you understand what you have read and practice proving things. Homework will not be collected or graded, but the short exam questions will be very similar (or identical) to homework from that week.

**Course grade:** Your best three short exam scores contribute 20 points each to your course grade and the final exam contributes 40 points. Letter grades will correspond (approximately) to the following ranges:

Score	Grade
90% – 100%	A– to A+
79% – 89%	B– to B+;
60% – 78%	C to C+
50% – 59%	D
0% – 49%	F

**IMPORTANT SUMMER SESSION I DATES**

**Last day to enroll:** Wednesday, June 25.

**Last day to drop:** Sunday, June 29.

**Withdrawal period:** Monday, June 30 - Friday, July 11.

**Last day to change grade option:** Friday, July 4.

Note that during Summer Session there is no auditing of classes, no 'Add by Petition' and no 'Administrative Drop by Instructor.' Failure to attend class does not constitute a 'Drop' of the course. **WARNING:** if you enroll in a class, even for a moment, and then drop out of Summer Session completely, there is a **\$50 cancellation fee** that you must pay. All deadlines are final.

## TIPS FOR SUCCESS

- ★ Come to all the lectures, and come prepared — read the assigned sections at least once before the lecture, so you have an idea of what we will be discussing in the lecture. Take particular note of the *Definitions!*
- ★ Read the material again after the lecture, this time in more depth. Read actively: take notes, make a list of questions, etc. Try to sketch the proofs of the theorems in the book yourself and compare your proofs to the proofs in the book.
- ★ Work on the homework after the second reading. Make a note of the problems that you find difficult so that you can ask about them.
- ★ Take advantage of office hours.
- ★ Form study groups. Technical skills (e.g., computing integrals or derivatives) can be practiced alone, but concepts and proofs need to be *discussed*.
- ★ You can expect to spend 15-25 hours per week on this material outside of class.  
To be successful, you should spread your studying over the week. Studying for five, six or more hours in a row is not as effective as blocks of 1 - 2 hours with breaks.
- ★ If you feel that you are getting lost, take action. Come to office hours to clear up any confusion or difficulty. Moreover, you should come to office hours *prepared* — the more specific your questions the more specific and helpful the answers will be.

**CHEATING:** Cheating in any form (e.g., using cheat sheets on exams) will not be tolerated. Any student caught cheating will be reported to the Mathematics department and to his or her college provost. In almost all cases, students caught cheating will receive a failing grade. Students who help others cheat are also considered cheaters.

Cheating devalues everybody's grades, and you should not tolerate it either.

## Lecture Schedule and Exam Dates.

**Monday, 6-23:** Introduction. The field axioms and the rational numbers; Ordered sets and ordered fields; least upper bounds and greatest lower bounds.

**Reading.** Sections 1.1 - 1.18.

**Homework.** Chapter 1: 1, 2, 3, 4.

**Wednesday, 6-25:** The *least-upper-bound* property; The field  $\mathbb{R}$ ; Euclidean spaces.

**Reading.** Sections 1.19 - 1.23, 1.35 - 1.38, Appendix.

**Comment:** In theorem 1.35, you should may assume that  $a_1, \dots, a_n$  and  $b_1, \dots, b_n$  are *real numbers*. For a real number  $x$ , we have  $\bar{x} = x$ .

**Homework.** Chapter 1: 5, 6, 7, 15, 18.

**Comment:** Problems 6 and 7 require a lot of work, I don't expect you to finish them in one go. But you should keep plugging away at them.

**Friday, 6-27:** Short review of set theory; Metric spaces. ***Exam 1***

**Reading.** Sections 2.1 - 2.17.

**Homework.** Chapter 2: 1, 2, 3, 4, 11.

**Monday, 6-30:** Metric topology.

**Reading.** Sections 2.18 - 2.30.

**Homework.** Chapter 2: 5, 6, 7, 8, 9.

**Wednesday, 7-2:** Compact sets.

**Reading.** Sections 2.31 - 2.42.

**Homework.** Chapter 2: 10, 12, 13, 14, 15.

**Friday, 7-4:** ***Holiday - no class***

**Reading.** Sections 2.1 - 2.42.

**Monday, 7-7:** Perfect sets and Connected sets. ***Exam 2***

**Reading.** Sections 2.43 - 2.47.

**Homework.** Chapter 2: 17, 18, 19, 21, 22, 29.

**Comment:** You should also look at problems 23 - 25. Their conclusions are useful.

**Wednesday, 7-9:** Sequences, subsequences and Cauchy sequences;  $\limsup$  and  $\liminf$ .

**Reading.** Sections 3.1 - 3.18.

**Homework.** Chapter 3: 1, 2, 3, 5, 20.

**Friday, 7-11:** Special sequences; Series. ***Exam 3***

**Reading.** Section 3.19 - 3.29.

**Homework.** Chapter 3: , 6 (a, b, c), 7, 8.

**Monday, 7-14:** The number  $e$ ; Power series; The root and ratio tests; Summation by parts; Absolute convergence; Cauchy products.

**Reading.** Sections 3.30 - 3.50. (and *skim* the rest of the chapter).

**Homework.** Chapter 3: 9, 10, 13, 16, 17.

**Wednesday, 7-16:** Continuous functions I.

**Reading.** Sections 4.1 - 4.16.

**Homework.** Chapter 4: 1, 2, 3, 4, 5.

**Friday, 7-18:** Continuous functions II.      ***Exam 4***

**Reading.** Sections 4.17 - 4.31 (and *skim* the rest of the chapter).

**Homework.** Chapter 4: 8, 11, 13, 14.

**Comment:** You should also study problems 20 - 22. They establish a very important property of the metric topology.

**Monday, 7-21:** Differentiable functions I.

**Reading.** Sections 5.1 - 5.11.

**Homework.** Chapter 5: 1, 2, 3, 4, 7.

**Wednesday, 7-23:** Differentiable functions II.

**Reading.** Sections 5.12 - 5.15.

**Homework.** Chapter 5: 9, 21, 22, 23.

**Friday, 7-25:**                      **FINAL EXAM** (in class).