Math 23A (Online)
Vector Calculus
University of California Santa Cruz

Instructors and Course Creators
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Course Description (from the UCSC course catalog)
Vectors in n-dimensional Euclidean space. The inner and cross products. The derivative of functions from n-dimensional to m-dimensional Euclidean space is studied as a linear transformation having matrix representation. Paths in 3-dimensions, arc length, vector differential calculus, Taylor’s theorem in several variables, extrema of real-valued functions, constrained extrema and Lagrange multipliers, the implicit function theorem, some applications.

Pre-Requisites, Entrance Requirements and Education Code
Prerequisite(s): course 19B or 20B or AP calculus BC exam score of 4 or 5.
General UCSC Education Code: MF (Mathematical and Formal Reasoning.)

Course Learning Objectives

• Master the differential calculus of several variables at a high level.
• Topics to be mastered are the nature of the derivative as a matrix, vectors, vector fields, applications to physics and the theory of extrema including Lagrange Multipliers.

Learning Activities
The course uses a learning management system (LMS) to organize content and act as a portal to other platforms including an interactive e-textbook, a homework platform with immediate feedback, and a discussion forum. There are ten one-week modules in the course that can be reconfigured to accommodate a shorter summer session version of
the course. Each module consists of 1-3 lessons. Each lesson consists of four learning activities:

1. Reading from textbook
2. View online video lectures
3. Complete homework
4. Interact with classmates, TA’s, instructor through online discussion boards

To succeed in this course, students must actively engage in all four activities.

1. Readings
Each lesson indicates the required reading and links to the interactive e-textbook based on two textbooks: Rogawski, *Calculus, Early Transcendentals, 2nd ed.* and Marsden/Tromba, *Vector Calculus, 6th ed.* As students read each section in the e-book, they encounter graded progress-check questions as well as supplemental practice exercises. Progress-check questions provide immediate feedback and are set up to allow three attempts.

2. Videos
Each lesson also contains a number of short video lectures (~5-15 minutes each) that explain in greater detail the concepts and techniques for the lesson and provide examples. Students can pause or rewind lecture videos if they don’t understand content covered as well as watch the videos at a reduced or accelerated speed, based on their personal preferences. They can also come back for review at any time. Students are encouraged to alternate between viewing the videos and reading the corresponding sections in the e-book.

3. Homework
Each lesson links to the required homework assignment on the online homework system. This system provides immediate feedback on students’ answers to each problem and resets the problems to allow students to try again. Students are allowed unlimited attempts. In preparation for exams, students have access to optional, ungraded practice sets.

4. Interaction
The course offers students an online discussion forum for students to post questions relating to the video lectures, homework, reading, and course logistics. Students are encouraged to respond to each other’s questions, and instructors and TA’s monitor
these forums, as well, responding to student questions as well. In addition, the teaching staff holds regular online and in-person office hours, as well as optional drop-in sections. Details on how to participate are outlined on the lesson pages and other dedicated pages in the LMS.

**Course Modules.**

**Get Started**
- **Quick Start Guide** (Essential course information on getting connected to course tools and getting help with logistical, content and technical issues)
- **Syllabus** (Table view of due dates and syllabus with course policies)
- **Exam Information** (Dates and expectations)
- **Academic Integrity** (Campus academic integrity rules and policies)
- **Course Contract** (Pledge to abide by academic integrity policy)

**Support Options**
- **Optional Discussion Sections**
- **Office Hours** (Schedule and information on participating in online and in-person office hours)
- **Tutoring Options**
- **Study Groups**
- **Disability Accommodations**
- **Study Tips**

*To access the course materials, students must complete the course contract by the end of the first week of instruction*

**Module 1: Vectors in Two- and Three-Dimensional Space**

**Learning Objectives:**
- To master the notions of bound and free vector, the dot product of vectors, the length of a vector and the angle between vectors
- To be able to calculate equations of lines in space

**Topics:**
- Lesson 11.1: Vectors in Two- and Three-Dimensional Space
- Lesson 11.2: the Inner or Dot Product, Length and Distance
Module 2:
Learning Objectives:
• To master the concept of a 2x2 and a 3x3 matrix and its determinant
• To understand the cross product of vectors.
• To be able to calculate equations of planes in space
• To understand polar, cylindrical and spherical coordinates.
Topics:
• Lesson 11.3: Matrices, Determinants and the Cross-Product
• Lesson 11.5: Polar, Cylindrical and Spherical Coordinates

Module 3
Learning Objectives:
• To understand the classical quadratic surfaces in 3-space
• To master the idea of n-dimensional space
• To understand the concept of an nxm matrix and an nxn determinant
Topics:
• Lesson 11.6: A Survey of Quadratic Surfaces
• Lesson 11.7: n-Dimensional Space Lesson 2.7: Limits at Infinity

Module 4
Learning Objectives:
• To master the concepts of paths and curves in space
• To be able to calculate the length of a curve in space
Topics:
• Lesson 12.1: Paths and Curves
• Lesson 12.2: Acceleration
• Lesson 12.3: Arc Length

Module 5
Learning Objectives:
• To master the concept of limit and continuity of functions of several variables
• To understand the graph of a real-valued function of several variables
Topics:
• Lesson 13.1: The Geometry of Real-Valued Functions
• Lesson 13.2: Limits and Continuity
• MATH 23A Midterm Review

Module: Midterm Practice Materials
Module 6
Learning Objectives
• To master the concept of the derivative of real valued and vector valued functions of several variables
• To be able to calculate partial derivatives of functions of several variables
• To be able to calculate tangent planes to graphs
Topics:
• Lesson 13.3: Differentiation
• Week 6 Survey <mid-course feedback for formative evaluation>

Module 7
Learning Objectives
• To master the properties of the derivative, in particular the chain rule
• To understand the gradient vector
• To be able to understand and calculate the directional derivative
• To be able to calculate tangent planes to surfaces
Topics:
• Lesson 13.4: Properties of the Derivatives
• Lesson 13.5: Gradients and Directional Derivatives

Module 8
Learning Objectives
• To master higher derivatives and Taylor's Theorem
• Gain an understanding about the equality of mixed partials
Topics:
• Lesson 14.1: Iterated Partial Derivatives
• Lesson 14.2: Taylor's Theorem

Module 9
Learning Objectives:
• To master the theory of extrema of real valued functions of several variables
• To understand the method of Lagrange Multipliers
Topics:
• Lesson 14.3: Extrema of Real-Valued Functions
• Lesson 14.4: Constrained Extrema and Lagrange Multipliers

Module 10
Learning Objectives
• To master the concept of vector field
• To understand the physical significance of the curl and divergence of vector fields

Topics:
• Lesson 14.6: Vector Fields
• Lesson 14.7: The Divergence and Curl of a Vector Field
• Final Student Poll

Module: Final Exam Review
• Exam Information
• Practice Finals
• Practice Problems for Final

Course Resources
• Course Overview
• Instructor Introductions

Instructional Strategy

<table>
<thead>
<tr>
<th>Instructional Element</th>
<th>Intended Learning Experience</th>
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<tbody>
<tr>
<td>Canvas</td>
<td>• Primary course portal students use to view videos and find general information and announcements on the course. Faculty update course information as necessary.</td>
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<td>• Canvas is used by students for every study session – students access the ebook, discussion forum, and homework portal through Canvas</td>
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<td>• Engagement can be evaluated through course analytics that provide data on how frequently students login, number of pages visited, etc.</td>
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<td>• Single-sign on from within Canvas to all the various resources</td>
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<td>Instructional Element</td>
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<td>creates a seamless and highly cohesive experience for students.</td>
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<td>• The main grade book will be maintained in this platform via deep integration between Canvas and LaunchPad (see below for information on LaunchPad), and final grades for all students are calculated in Canvas and then exported to the appropriate UC Student Information System(s)</td>
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<td>• Portability allows students to view videos from mobile devices.</td>
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**Pedagogy:** Canvas is a central hub through which students access all course information and communicate with teaching staff and peers. It provides organization and structure to the course.
| Launchpad | • Online learning platform that contains the e-textbook.  
• Offers interactive reading assignments for each lesson.  
• Course designers customize the e-book by editing passages and/or deleting extraneous material, embedding additional interactive features, creating progress check questions, etc.  
• Testing platform for students who take exams online.  
• Faculty engage regularly with LaunchPad to create exam questions, grade exams, etc.  
• Engagement and performance can be assessed through analytics that provide data on how frequently students login, scores on progress check questions, minutes spent, etc.  

Pedagogy: Customized E-Book provides students with a text that is closely associated with assessments and other course content. Interactive elements engage students and lead to greater comprehension of reading. Portability allows students to view textbook from mobile devices. LaunchPad allows students to test their understanding of course content frequently. It provides immediate feedback on homework questions and hence reinforces knowledge and flags areas that need further study or exploration. |
|---|---|
| Piazza | • Discussion platform that allows students to interact with classmates, teaching assistants and faculty.  
• Teaching staff will facilitate and provide regular presence by answering questions and endorsing helpful student posts.  
Teaching staff will also create tags and folders to organize posts, and provide seed questions to stimulate discussion.  
• Engagement can be assessed through analytics that provide data on how frequently a student logins and how frequently they post questions and answers.  

Pedagogy: Discussion boards lead to less isolation in online courses, provide students with opportunities to collaborate and form community, and allow for much quicker response times to student questions. By responding to classmates’ questions, students have an opportunity to demonstrate knowledge and explain complicated concepts in their own words, thus leading to greater understanding and comprehension. Students can post anonymously, allowing more students to be comfortable asking questions. |
| Video Lectures / MediaCore | • Online video hosting platform.  
• Engagement will be assessed through analytics showing number and duration of video views. |
<table>
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<tr>
<th>Pedagogy: Online lecture and course videos provide additional explanation and illustration of complex material. Permits students to review (rewind) lectures on demand. Supports anytime, anywhere learning. Insures a sense of instructor presence,</th>
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</table>
| • Offered both online and in-person several times a week by Faculty and TAs. One-on-one appointments available.  
• Virtual Office Hours conducted using Adobe Connect, a web-conferencing platform that supports real-time audio and video conferencing, text-based chat, two-way screensharing and whiteboarding that allows students as well as faculty to write equations, screen sharing, and video and powerpoint presentation. |
| Office Hours/Adobe Connect |
| Pedagogy: Office Hours give both remote and local students an opportunity to connect with teaching staff and peers and get individualized instruction on content with which they are struggling. |
| • Offered regularly during the week – extended meeting period where students can drop in to work face-to-face with other students and TAs. TAs may provide mini-lectures on concepts covered in weekly modules. Extra meetings scheduled during exam periods.  
• Online versions of sections are offered by teaching staff for remote students, using Adobe Connect, a web-conferencing platform that supports real-time audio and video conferencing, text-based chat, two-way screensharing and whiteboarding that allows students as well as faculty to write equations, screen sharing, and video and powerpoint presentation. |
| Drop-in Sections |
| Pedagogy: provides an additional opportunity for interaction and small group instruction. |
| • Face-to-Face Tutoring available at UCSC through MSI (Modified Supplemental Instruction, a campus program), Math Department, and ACE (a divisional program for students of underrepresented groups). |
| Tutoring |
| Pedagogy: provides students with additional support if they are struggling with comprehension of the material. Can be used by both lower and higher performing students. |
| • Informal student-led study group set up for each residential college and/or remote campus.  
• One to three student volunteer facilitators appointed.  
• Groups are encouraged to meet in person or online at least once |
| College-Based Study Groups |
Assessment Activities

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<tr>
<th>Component</th>
<th>Description</th>
<th>Percentage</th>
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| Homework  | Online Homework problems for each lesson accessed through Launchpad  
• Low stakes questions with unlimited attempts and immediate feedback in LaunchPad.  
• Students can check their understanding of the content in each lesson and are incentivized to complete homework through points earned toward final grade. | 15%        |
| Quizzes   | Periodic online quizzes in LaunchPad  
• The online quizzes are distinguished from on-line homework by being limited in time and by the absence of hints and feedback.  
• Students have only one attempt on each question.  
• There will be partial credit (where appropriate). TAs and instructors will check answers and may assign partial credit after the computer score has been calculated.  
• A final score on a quiz or other on-line test may be higher than what students receive immediately after submitting the test to LaunchPad. | 10%        |
| Reading   | Reading and progress-check questions in Launchpad  
• Low stakes progress-check questions allowing students 3 attempts/question. | 5%         |
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<tr>
<td>Proctored Midterm</td>
<td>Students may elect to take their proctored exam either online or on-campus, in-person (online exams proctored by third-party vendor, ProctorU)</td>
<td>30%</td>
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<td>• Encourages students to complete the readings and gives them an opportunity to check their understanding.</td>
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<td>• Formative assessments that give students’ an opportunity to evaluate progress and make improvements where necessary.</td>
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<td>• Provides students with examples of how questions are formatted on final.</td>
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<td>Final Exam</td>
<td>Comprehensive final exam. Online or in-person administration options (online exams proctored by Proctor U)</td>
<td>40%</td>
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<td>• Summative assessment used to evaluate student learning.</td>
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<tr>
<td>Total:</td>
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<td>100.00%</td>
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