Math 110 - Introduction to Number Theory
Summer 2023 - Session 1
Synchronous Online

Course Information

Zoom Link Available on Canvas
MWF 9:00am - 11:30am

Instructor Information

Malachi Alexander
Email: majalexa@ucsc.edu

Instructor Office Hours

Zoom Link Available on Canvas
F 12:00pm-1:00pm

Teaching Assistant

Justin Lake
Email: jlake@ucsc.edu

TA Office Hours

Zoom Link Available on Canvas
T 11:00 AM-12:00 PM

You may request an appointment with the instructor or the teaching assistant if the times provided does not work. To request, send an email with subject line "Math 110 Office Hours Request" and provide two or three times which work best for you. In general, we will not regularly check emails during weekends, so we cannot guarantee you responses during weekend days.

Additional Readings

Elementary Number Theory by Underwood Dudley

No textbook is required for this course.

Course Overview

Number theory is a subject which is characterized by the problems it attempts to understand. It both draws techniques and problems from a wide range of subjects and serves as a tool to solve problems in these subjects. In this course, we establish some fundamental notions which serve as a starting point in investigating this beautiful and vast subject.

This course has been separated into five modules.

1: Linear Diophantine Equations

2: Unique Factorization

3: Modular Congruences

4: Arithmetic Functions

5: Number Systems

Participation

This course will be taught synchronously; however, the structure of this course is flipped and *can be completed asynchronously without penalty*. In this format, you will be expected to read and watch pre-recorded lectures (found in each Module on Canvas) outside of the class hours and class hours will instead be focused on problem-solving and collaboration. Although attendance to lecture is not mandatory, it is great opportunity to explore topics, ask questions and collaborate with your peers! I hope this flexibility assists you in your success!

Grading

In this course, we will be using *mastery-based grading* to determine final letter grades. This means that assignments will be scored on a satisfactory/unsatisfactory scale; if an 'unsatisfactory' score is designated on an assignment, you will be provided with feedback and will be given an opportunity to resubmit the work for regrading as long as it is submitted by the deadline. Each module consists of several smaller concepts to master which will count towards module completion.

There will be a portfolio project which aims to showcase your work throughout the course. See Final Project below for more details. To pass the course, you must complete the Final Project.

Late & Regrade Policy
Each student will be given 10 opportunities to submit problems late for feedback without any questions asked or ask for more feedback after a second 'unsatisfactory' score on an assignment. At the end of the course, any student with remaining opportunities will have their grade boosted as listed below. *Late assignments will always be graded without penalty; however, you will not be given written feedback and it may be graded at a later date.*

Use the following to determine your final letter grade in the course:

A+ Meet requirements of **A** and 4 or more regrade opportunities.

A Obtain 90% mastery on all modules and receive a satisfactory on the final project.

A- Meet requirements of **B** and 7 or more regrade opportunities.

B+ Meet requirements of **B** and 4 or more regrade opportunities.

B Obtain 80% mastery on 4 out of 5 modules.

B- Meet requirements of **C** and 7 or more regrade opportunities.

C+ Meet requirements of **C** and 4 or more regrade opportunities.

C Obtain 70% mastery on 3 out of 5 modules.

C- Meet requirements of **D** and 7 or more regrade opportunities.

D+ Meet requirements of **D** and 4 or more regrade opportunities.

D Obtain 60% mastery on 2 out of 5 modules.

F For any level of completion under requirements to obtain a **D**.

Learning Outcomes

AI1: Develop connections between algebra, analysis, geometry and number theory and use these various perspectives to develop intuition and solve number theoretic problems.

HC1: Learn to collaborate through problem-solving, providing and receiving feedback, and communicating logical arguments to peers.

HC2: Reflect on one's personal journey and identity in mathematics, and relate these to the historical context of the subject, and one's current career path options and personal interests in mathematics, programming and teaching.

LHtL1: Developing professional skills related to pursuing mathematics, including reading mathematical texts, LaTeX typesetting skills for professional writing, techniques for internalizing important definitions and theorems, and tailoring communication of mathematics to different audiences.

Modules

Module 1: In this module, we will reinforce understanding of divisibility, greatest common divisor, and least common multiple, learn the Euclidean algorithm and apply each of these concepts to find the general solution of a linear Diophantine equation with at least two variables.

Module 2: In this module, we will build understanding of primes, irreducibles, and unique factorization over the integers and apply unique factorization to notions of divisibility, greatest common divisor, and least common multiple. In addition, we will use algorithms to generate small primes and develop intuition of the distribution of primes.

Module 3: In this module, we will revisit the notion of modular congruence, develop a working knowledge of common modular relationships, and apply this knowledge to solve systems of linear congruence equations, quadratic congruence equations. We will also learn about quadratic reciprocity, Legendre's symbol and its relation to congruence.

Module 4: In this module, we will learn about arithmetic functions such as the divisor function and Euler totient function, various special properties of these special functions and the Möbius inversion formula.

Module 5: In this module, we will develop insight into rational, irrational, algebraic, transcendental and other special numbers in the complex number system.

Assignments

Syllabus Quiz: At the beginning of the course, you will be required to complete a quiz checking that you understand this document.

Mathematical Autobiography: At the beginning of the course, you will be required to write a short autobiography detailing your experience with mathematics. This is designed for me to get to know you better and understand your needs for this course.

Knowledge Checks: In each of the modules, there will be several quizzes which check your understanding at various points in the reading. These are designed to be quick check-ins to see if you understand definitions and how to do simple computations related to the readings. These will be used to determine mastery for less involved mastery requirements.

Problem Sets: In each of the modules, you will be assigned problems which test your understanding on how to use the definitions and concepts to solve problems. These do not have to be submitted in LaTeX however, it is recommended. These will be used to determine mastery for more involved mastery requirements.

All written problems must be submitted in PDF format. Templates will be provided for LaTeX documents.

Final Project: In each of the modules, you will be given options to write a formal proof (in LaTeX), write a program for algorithms presented in the course (or a computation problem), or write a lesson plan for how you would teach a particular concept to a specific group of students. See Final Project assignment on Canvas for details.

There is no final exam for this course.

Course Schedule

Week	Monday	Wednesday	Friday
1	Section 2.1-2.3	Section 2.4-2.5	Section 2.6-2.7
2	Section 3.1-3.2	Section 3.3-3.5	Section 4.1
3	Section 4.2-4.3	Section 4.4-4.5	Section 4.5-4.7
4	Section 5.1-5.3	Section 5.4-5.6	Section 5.7-5.8
5	Section 6.1-6.3	Section 6.4-6.5	No Lecture

Section numbers are in reference to the course notes.

Student Resources

Disability Resource Center
UC Santa Cruz is committed to creating an academic environment that supports its diverse student body. If you are a student with a disability who requires accommodations to achieve equal access in this course, please affiliate with the DRC. I encourage all students to benefit from learning more about DRC services to contact DRC by phone at 831-459-2089 or by email at drc@ucsc.edu. For students already affiliated, make sure that you have requested Academic Access Letters, where you intend to use accommodations. You can also request to meet privately with me during my office hours or by appointment, as soon as possible. I would like us to discuss how we can implement your accommodations in this course to ensure your access and full engagement in this course.

Title IX
The Title IX Office is committed to fostering a campus climate in which members of our community are protected from all forms of sex discrimination, including sexual harassment, sexual violence, and gender-based harassment and discrimination. Title IX is a neutral office committed to safety, fairness, trauma-informed practices, and due process.

Title IX prohibits gender discrimination, including sexual harassment, domestic and dating violence, sexual assault, and stalking. If you have experienced sexual harassment or sexual violence, you can receive confidential support and advocacy at the Campus Advocacy Resources & Education (CARE) Office by calling 831-502-2273. In addition, Counseling & Psychological Services (CAPS) can provide confidential, counseling support, 831-459-2628. You can also report gender discrimination directly to the University's Title IX Office, 831-459-2462. Reports to law enforcement can be made to UCPD, 831-459-2231 ext. 1. For emergencies call 911.

Counseling and Psychological Services
Many students at UCSC face personal challenges or have psychological needs that may interfere with their academic progress, social development, or emotional well-being. The university offers a variety of confidential services to help you through difficult times, including individual and group counseling, crisis intervention, consultations, online chats, and mental health screenings. These services are provided by staff who welcome all students and embrace a philosophy respectful of clients' cultural and religious backgrounds, and sensitive to differences in race, ability, gender identity and sexual orientation.

Student Success and Engagement Hub
The Division of Student Success provides campus-wide coordination and leadership for student success programs and activities across departments, divisions, the colleges, and administrative units.

Tutoring and Learning Support
At Learning Support Services (LSS), undergraduate students build a strong foundation for success and cultivate a sense of belonging in our Community of Learners. LSS partners with faculty and staff to advance educational equity by designing inclusive learning environments in Modified Supplemental Instruction, Small Group Tutoring, and Writing Support. When students fully engage in our programs, they gain transformative experiences that empower them at the university and beyond.

Slug Support Program
College can be a challenging time for students and during times of stress it is not always easy to find the help you need. Slug Support can give help with everything from basic needs (housing, food, or financial insecurity) to getting the technology you need during remote instruction. To get started with SLUG Support, please contact the Dean of Students Office at 831-459-4446 or you may send us an email at deanofstudents@ucsc.edu.

Slug Help/Technology
The ITS Support Center is your single point of contact for all issues, problems or questions related to technology services and computing at UC Santa Cruz. To get technological help, simply email help@ucsc.edu.

On-Campus Emergency Contacts
For all other help and support, including the health center and emergency services, start here. Always dial 9-1-1 in the case of an emergency.